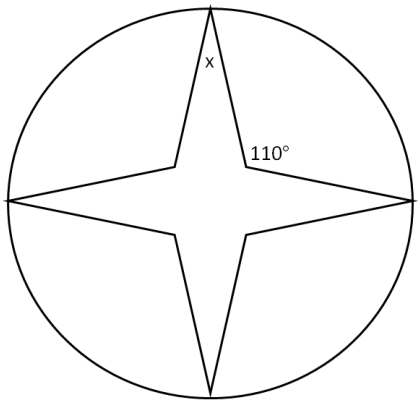
IMO Geometry Questions

Level: Intermediate Ref No: M01 **Puzz Points: 10**

The “star” octagon shown in the diagram is beautifully symmetrical and the centre of the star is at the centre of the circle. If angle NAE = 110°, how big is the angle DNA?



Level: Intermediate Ref No: M03 **Puzz Points: 10**

A quadrilateral ABCD has sides AB, BC, CD, DA of length x, y, z and t, respectively. The diagonals AC and BD cross at right angles. Prove that:

$$x^2 + z^2 = y^2 + t^2$$

Level: Intermediate Ref No: M08 **Puzz Points: 15**

Triangle ABG has a right angle at B.

Points C and E lie on side AG and points D and F lie on side BG so that the six line segments AB, BC, CD, DE, EF and FG are equal in length. Calculate the angle AGB.

Level: Intermediate Ref No: M12 **Puzz Points: 18**

The triangle ABC is right-angled at A, with AB = 6cm and AC = 8cm. Points X and Y are situated on BC such that AB = AY and AX = XC. Two isosceles triangles ABY and AXC are thus created. These triangles overlap, forming the region AXY. Calculate the area of this region.

Level: Intermediate Ref No: M13 **Puzz Points: 20**

A quadrilateral is enclosed by four straight lines with equations:

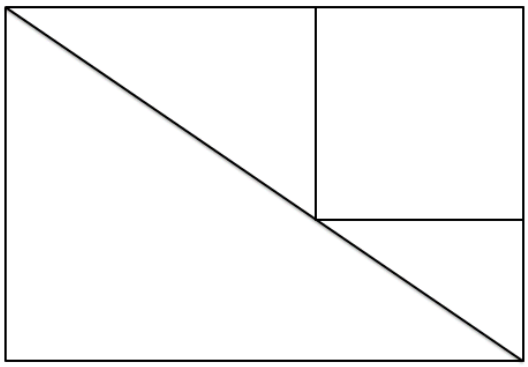
$$2y = x + 4$$
$$y = 2x - 4$$
$$2y = x - 2$$
$$y = 2x + 2$$

Calculate the area of this quadrilateral.

Level: Intermediate Ref No: M15 **Puzz Points: 20**

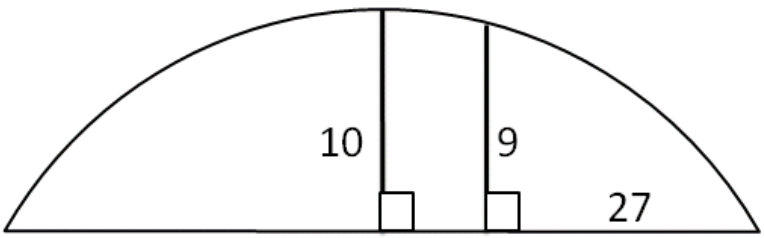
A square is constructed inside a rectangle of length *a* and width *b*, with the square touching the diagonal of the rectangle as shown in the diagram. If the square has side *h*, prove that:

$$\frac{1}{h} = \frac{1}{a} + \frac{1}{b}$$



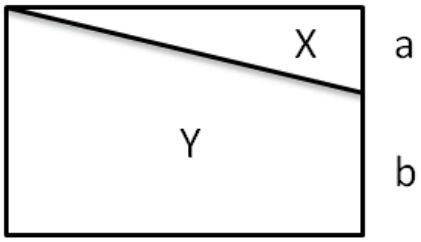
Level: Intermediate Ref No: M18 **Puzz Points: 23**

The cross section of a tunnel is a circular arc, as shown in the diagram. The maximum height of the tunnel is 10 feet. A vertical strut 9 feet high supports the roof of the tunnel from a point 27 feet along the ground from the side. Calculate the width of the tunnel at ground level.



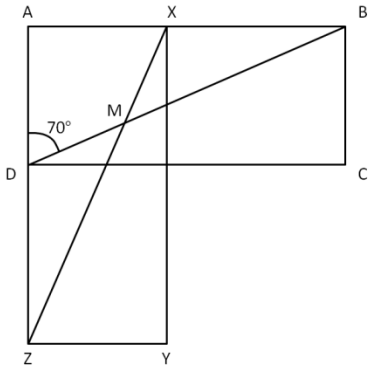
Level: Intermediate Ref No: M19 **Puzz Points: 10**

A rectangular piece of paper is cut into two pieces by a straight line passing through one corner, as shown. Given that area X : area Y = 2:7, what is the value of the ratio a:b?



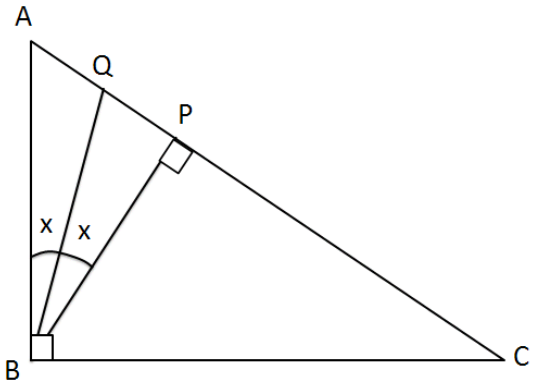
Level: Intermediate Ref No: M21 **Puzz Points: 10**

In the diagram, rectangles ABCD and AZYX are congruent, and $\angle ADB = 70°$. Find $\angle BMX$.



Level: Intermediate Ref No: M26 **Puzz Points: 15**

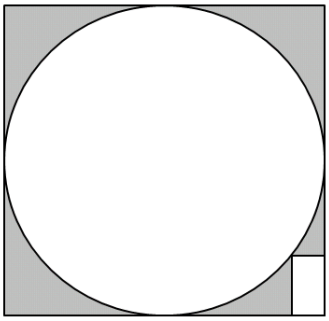
In triangle ABC, $\angle ABC$ is a right angle. Points P and Q lie on AC; BP is perpendicular to AC; BQ bisects $\angle ABP$. Prove that CB = CQ.



Level: Intermediate Ref No: M28 **Puzz Points: 15**

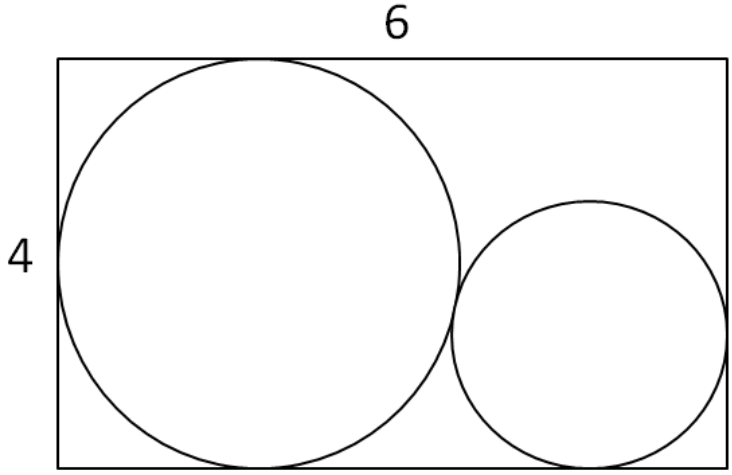
A circle is inscribed in a square and a rectangle is placed inside the square but outside the circle. Two sides of the rectangle lie along sides of the square and one vertex lies on the circle, as shown. The rectangle is twice as high as it is wide.

What is the ratio of the area of the square to the area of the rectangle?



Level: Intermediate Ref No: M33 **Puzz Points: 20**

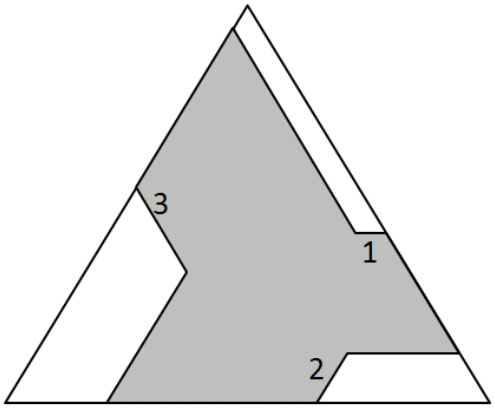
Two circles are drawn in a rectangle of 6 by 4, such that the larger circle touches three sides of the rectangle, whereas the smaller one only touches 2. Determine the radius of the smaller circle.



Level: Intermediate Ref No: M34 **Puzz Points: 20**

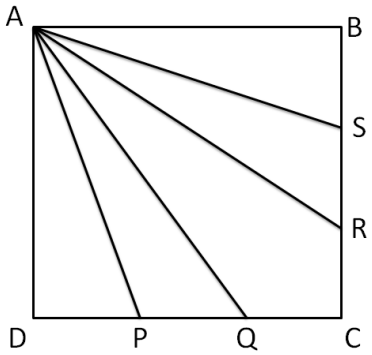
The nonagon shown shaded in the diagram has been made by removing three pieces from an equilateral triangle of side 12. All nine edges of the nonagon are parallel to sides of the triangle. Three edges have lengths 1, 2 and 3 as shown.

Calculate the length of the perimeter of the nonagon.



Level: Intermediate Ref No: M39 **Puzz Points: 10**

The diagram shows a square ABCD of side 10 units. Line segments AP, AQ, AR and AS divide the square into five regions of equal area, as shown. Calculate the length of QR.



Level: Intermediate Ref No: M40 **Puzz Points: 10**

How many right-angled triangles can be made by joining three vertices of a cube?

Level: Intermediate Ref No: M41 **Puzz Points: 13**

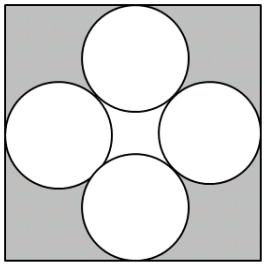
In a quadrilateral ABCD, AB = BC, $\angle BAC = 60°$, $\angle CAD = 40°$, AC and BD cross at X and $\angle BXC = 100°$.

Calculate $\angle BDC$.

Level: Intermediate Ref No: M45 **Puzz Points: 15**

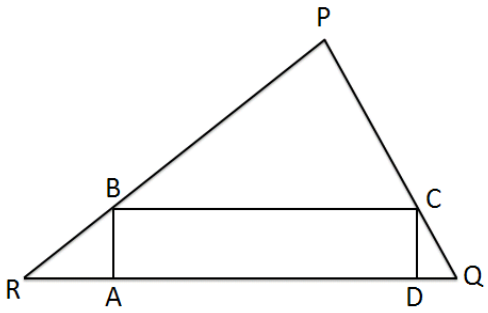
The diagram shows four circles of radius 1 placed inside a square so that they are tangential to the sides of the square at the midpoints of the sides, and to each other.

Calculate the shaded area.



Level: Intermediate Ref No: M47 **Puzz Points: 18**

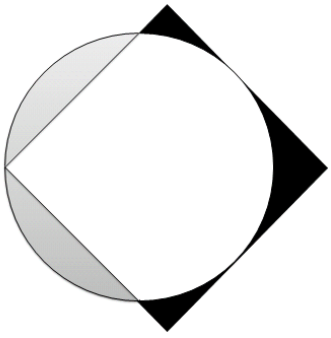
The diagram shows a rectangle ABCD inscribed inside a triangle PQR. The side, AB, of the rectangle is one third of the perpendicular height of the triangle from P to QR. What is the ratio of the area of the rectangle to the area of the triangle?



Level: Intermediate Ref No: M50 **Puzz Points: 20**

The diagram shows a circle of radius 2 and a square. The circle touches two sides of the square and passes through one corner of the square. The area of the region shaded black (inside the square but outside the circle) is X and the area of the region shaded grey (inside the circle but outside the square) is Y.

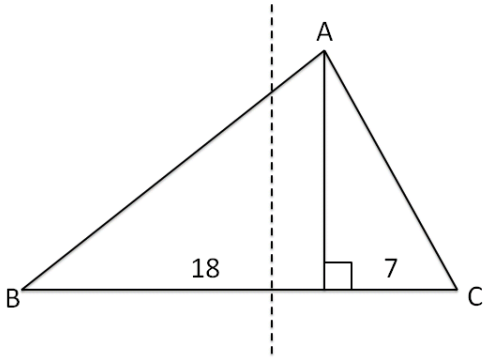
What is the value of $Y - X$?



Level: Intermediate Ref No: M52 **Puzz Points: 20**

The diagram shows a triangle in which the altitude from A divides the base, BC, in the ratio 18 : 7.

Find the ratio in which the base is divided by a line parallel to the altitude which cuts the triangle into two equal areas.



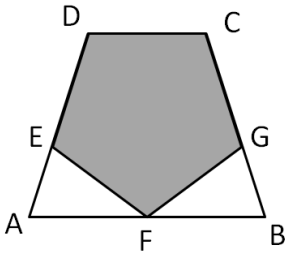
Level: Intermediate Ref No: M53 **Puzz Points: 23**

The coordinates of three vertices of a cube are (4, 0, 3), (6, 4, 1) and (2, 8, 5). Find the coordinates of a fourth vertex.

Level: Intermediate Ref No: M56 **Puzz Points: 10**

The diagram shows a regular pentagon $CDEFG$ inside a trapezium $ABCD$.

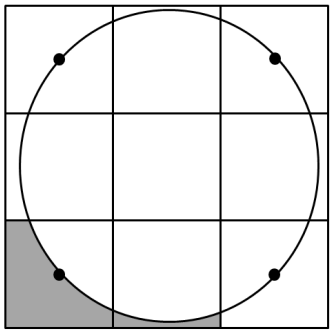
Prove that $AB = 2 \times CD$.



Level: Intermediate Ref No: M58 **Puzz Points: 10**

The diagram shows nine 1cm × 1cm squares and a circle. The circle passes through the centres of the four corner squares.

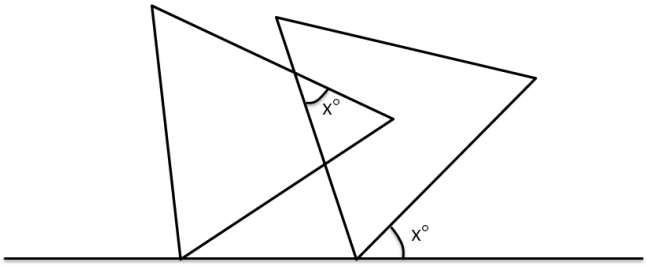
What is the area of the shaded region – inside two squares but outside the circle?



Level: Intermediate Ref No: M62 **Puzz Points: 15**

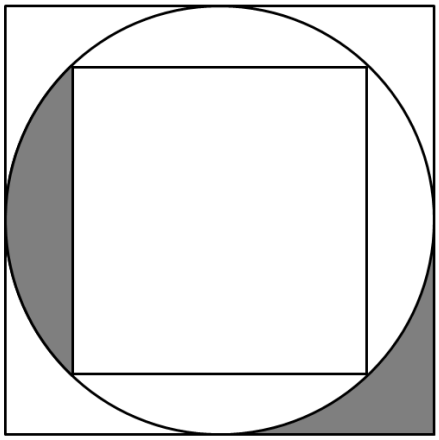
The diagram shows two equilateral triangles. The angle marked $x°$ are equal.

Prove that $x > 30$.



Level: Intermediate Ref No: M64 **Puzz Points: 15**

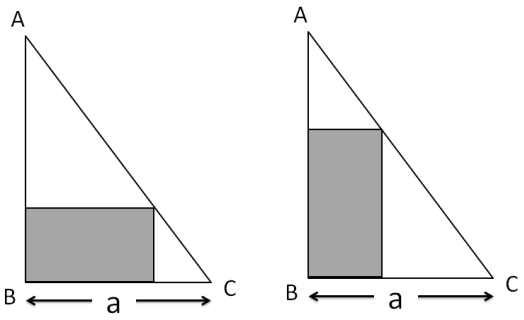
A square just fits within a circle, which itself just fits within another square, as shown in the diagram. Find the ratio of the two shaded areas.



Level: Intermediate Ref No: M69 **Puzz Points: 20**

The diagrams show a rectangle that just fits inside right-angled triangle *ABC* in two different ways. One side of the triangle has length *a*.

Prove that the perimeter of the rectangle is *2a*.

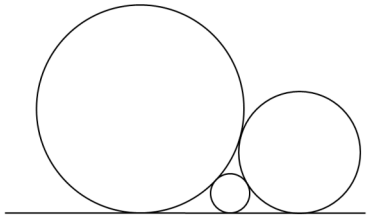


Level: Intermediate Ref No: M71 **Puzz Points: 23**

Three circles touch the same straight line and touch each other, as shown.

Prove that the radii *a*, *b*, and *c*, where *c* is smallest, satisfy the equation:

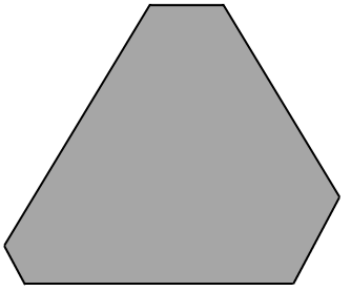
$$\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{c}}$$



Level: Intermediate Ref No: M74 **Puzz Points: 10**

A hexagon is made by cutting a small equilateral triangle from each corner of a larger equilateral triangle. The sides of the smaller triangles have lengths 1, 2 and 3 units. The lengths of the perimeters of the hexagon and the original triangle are in the ratio 5:7.

What fraction of the area of the original triangle remains?



Level: Intermediate Ref No: M75 **Puzz Points: 10**

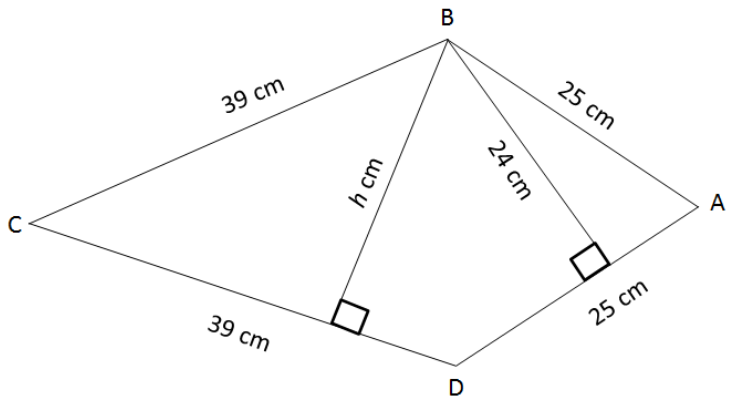
In the rectangle ABCD the midpoint of AB is M and AB : AD = 2 : 1. The point X is such that triangle MDX is equilateral, with X and A lying on opposite sides of the line MD.

Find the value of $\angle XCD$.

Level: Intermediate Ref No: M77 **Puzz Points: 13**

A kite has sides AB and AD of length 25cm and sides CB and CD of length 39cm. The perpendicular distance from B to AD is 24cm. The perpendicular distance from B to CD is *h* cm.

Find the value of *h*.



Level: Intermediate Ref No: M78 **Puzz Points: 13**

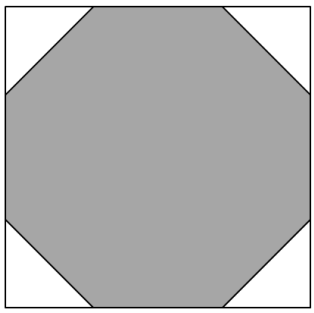
A regular tetrahedron ABCD has edges of length 2 units. The midpoint of the edge AB is M and the midpoint of the edge CD is N.

Find the exact length of the segment MN.

Level: Intermediate Ref No: M79 **Puzz Points: 15**

A regular octagon with sides of length *a* is inscribed in a square with sides of length 1, as shown.

Prove that $a^2 + 2a = 1$.



Level: Intermediate Ref No: M81 **Puzz Points: 15**

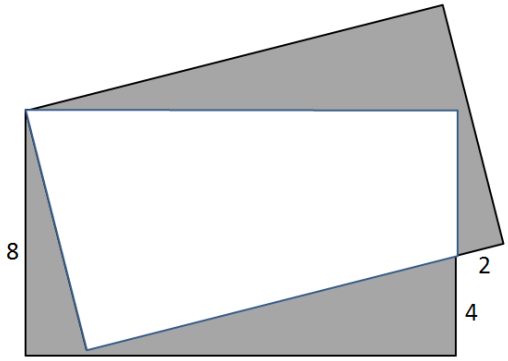
A triangle is bounded by the lines whose equations are $y = -x - 1$, $y = 2x - 1$ and $y = k$, where k is a positive integer.

For what values of k is the area of the triangle less than 2008?

Level: Intermediate Ref No: M82 **Puzz Points: 18**

Two congruent rectangles have a common vertex and overlap as shown in the diagram.

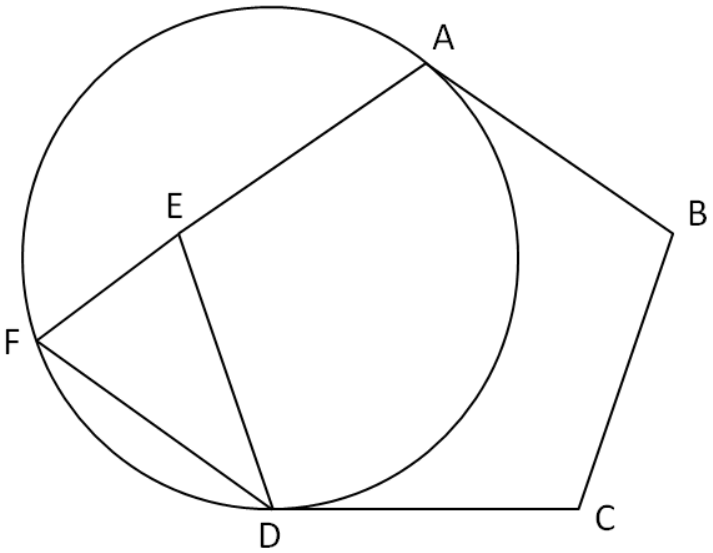
What is the total shaded area?



Level: Intermediate Ref No: M85 **Puzz Points: 20**

The diagram shows a regular pentagon ABCDE. A circle is drawn such that AB is a tangent to the circle at A and CD is a tangent to the circle at D. The side AE of the pentagon is extended to meet the circumference of the circle at F.

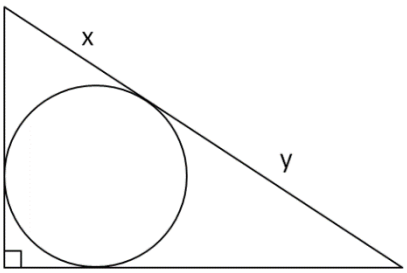
Prove that DE = DF.



Level: Intermediate Ref No: M87 **Puzz Points: 20**

A circle is inscribed in a right-angled triangle, as shown. The point of contact of the circle and the hypotenuse divides the hypotenuse into lengths x and y.

Prove that the area of the triangle is equal to xy.

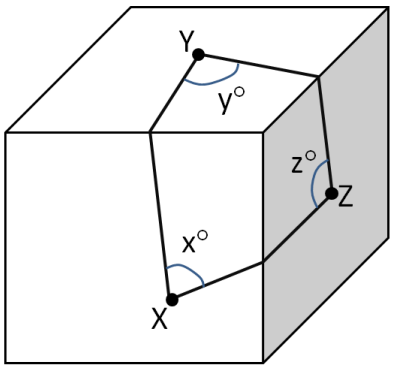


Level: Intermediate Ref No: M88 **Puzz Points: 23**

An ant lives on the surface of a cuboid which has points X, Y and Z on three adjacent faces.

The ant travels between X, Y and Z along the shortest possible path between each pair of points. The angles $x°$, $y°$ and $z°$ are the angles between the parts of the ant's path, as shown.

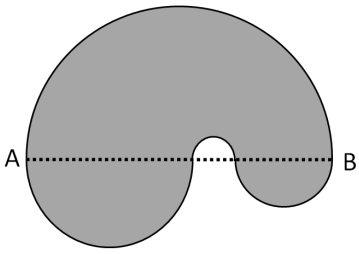
Prove that $x + y + z = 270$.



Level: Intermediate Ref No: M91 **Puzz Points: 10**

The boundary of a shaded figure consists of four semicircular arcs whose radii are all different. The centre of each arc lies on the line AB, which is 10cm long.

What is the length of the perimeter of the figure.



Level: Intermediate Ref No: M93 **Puzz Points: 10**

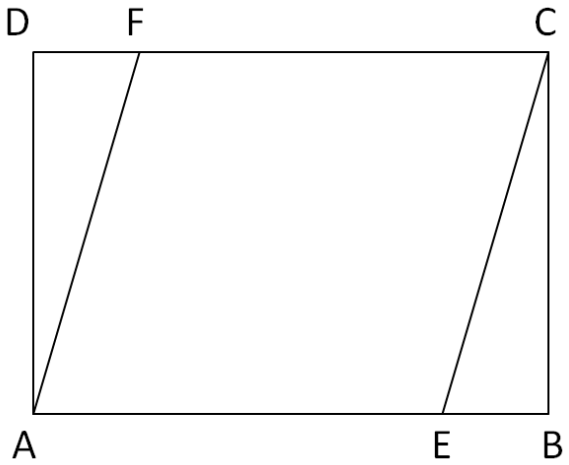
In a rectangle ABCD, the side AB has length $\sqrt{2}$ and the side AD has length 1. Let the circle with centre B and passing through C meet AB at X.

Find $\angle ADX$ (in degrees).

Level: Intermediate Ref No: M97 **Puzz Points: 15**

In the diagram, ABCD is a rectangle with $AB = 16cm$ and $BC = 12cm$. Points E and F lie on sides AB and CD so that AECF is a rhombus.

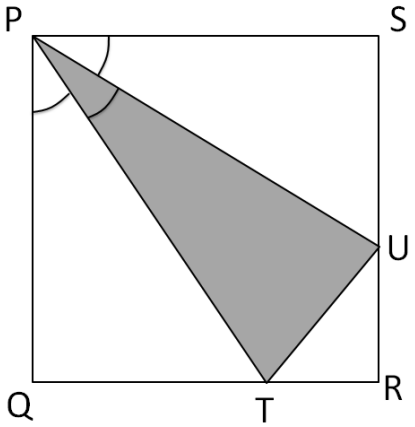
What is the length of EF?



Level: Intermediate Ref No: M99 **Puzz Points: 18**

The diagram shows a triangle PTU inscribed in a square PQRS. Each of the marked angles at P is equal to $30°$.

Prove that the area of the triangle PTU is one third of the area of the square PQRS.



Level: Intermediate Ref No: M104 **Puzz Points: 20**

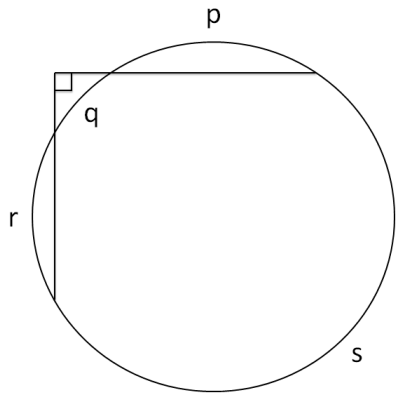
In a trapezium $ABCD$ the sides AB and DC are parallel and $\angle BAD = \angle ABC < 90°$. Point P lies on AB with $\angle CPD = \angle BAD$.

Prove that $PC^2 + PD^2 = AB \times DC$.

Level: Intermediate Ref No: M106 **Puzz Points: 23**

In the figure, p, q, r and s are the lengths of the four arcs which together form the circumference of the circle.

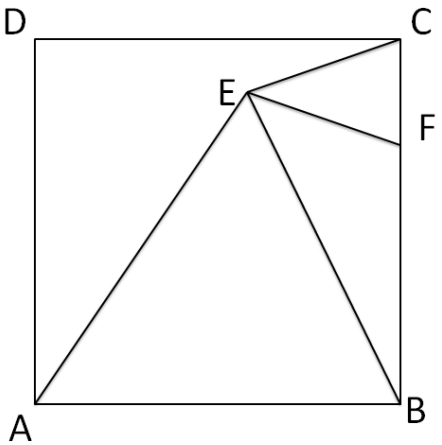
Find, in simplified form, an expression for s in terms of p, q and r.



Level: Intermediate Ref No: M108 **Puzz Points: 10**

The diagram shows a square ABCD and an equilateral triangle ABE. The point F lies on BC so that EC = EF.

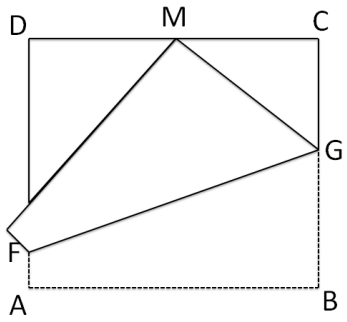
Calculate the angle BEF.



Level: Intermediate Ref No: M111 **Puzz Points: 13**

A square sheet of paper ABCD is folded along FG, as shown, so that the corner B is folded onto the midpoint M of CD.

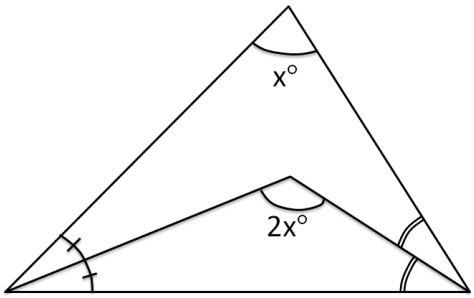
Prove that the sides of triangle GCM have lengths in the ratio $3:4:5$.



Level: Intermediate Ref No: M113 **Puzz Points: 15**

The diagram shows a triangle and two of its angle bisectors.

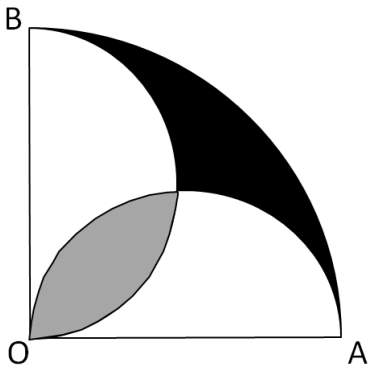
What is the value of *x*?



Level: Intermediate Ref No: M115 **Puzz Points: 15**

The diagram shows a quarter-circle with centre O and two semicircular arcs with diameters OA and OB.

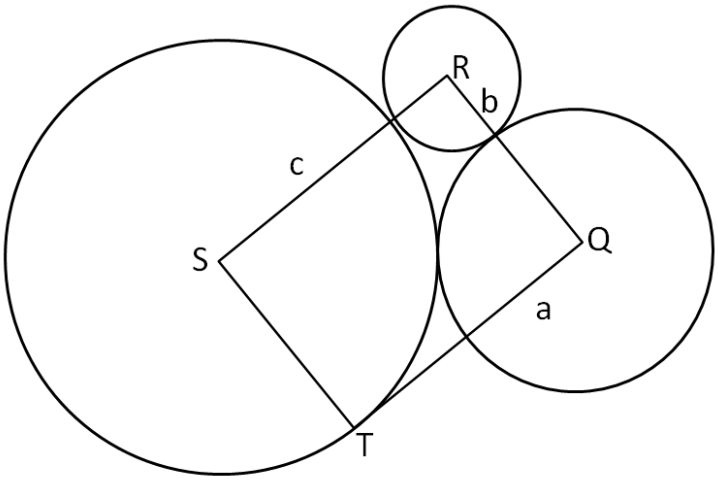
Calculate the ratio of the area of the region shaded grey to the area of the region shaded black.



Level: Intermediate Ref No: M116 **Puzz Points: 18**

The diagram shows three touching circles, whose radii are *a*, *b* and *c*, and whose centres are at the vertices Q, R and S of a rectangle QRST. The fourth vertex *T* of the rectangle lies on the circle with centre *S*.

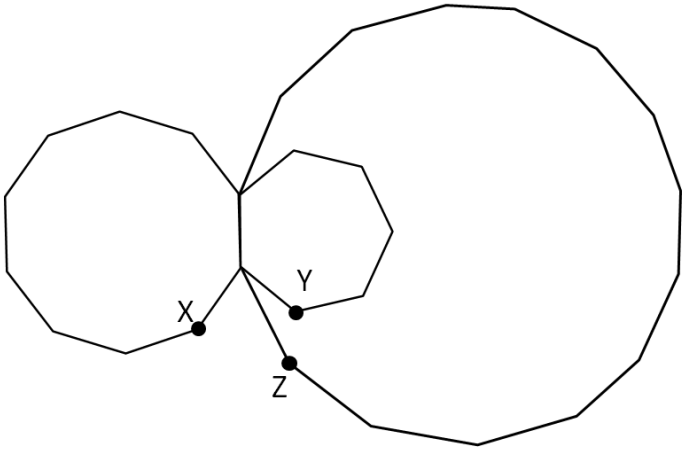
Find the ratio $a : b : c$.



Level: Intermediate Ref No: M119 **Puzz Points: 20**

The diagram shows a regular heptagon, a regular decagon and a regular 15-gon with an edge in common.

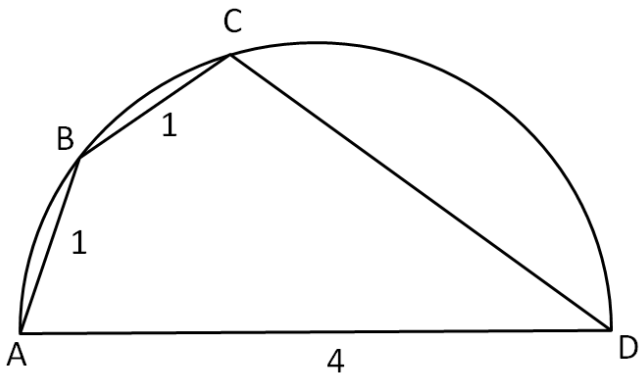
Find the size of angle *XYZ*.



Level: Intermediate Ref No: M121 **Puzz Points: 20**

The diameter *AD* of a circle has length 4. The points *B* and *C* lie on the circle, as shown, so that $AB = BC = 1$.

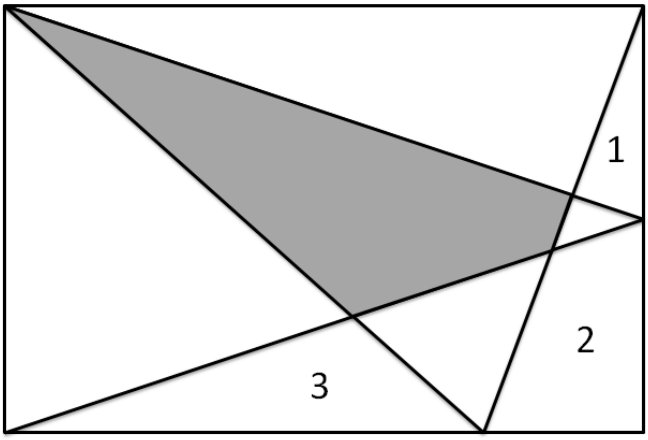
Find the length of CD.



Level: Intermediate Ref No: M122 **Puzz Points: 23**

The diagram shows a rectangle divided into eight regions by four straight lines. Three of the regions have areas 1, 2 and 3, as shown.

What is the area of the shaded quadrilateral?

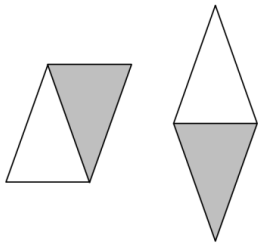


Level: Intermediate Ref No: M125 **Puzz Points: 10**

When two congruent isosceles triangles are joined to form a parallelogram, as shown in the first diagram, the perimeter of the parallelogram is 3cm longer than the perimeter of one of the triangles.

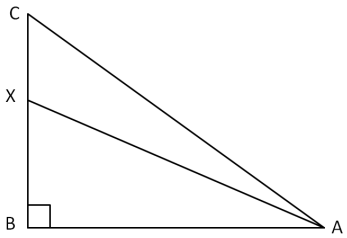
When the same two triangles are joined to form a rhombus, as shown in the second diagram, the perimeter of the rhombus is 7cm longer than the perimeter of one of the triangles.

What is the perimeter of one of the triangles?



Level: Intermediate Ref No: M126 **Puzz Points: 10**

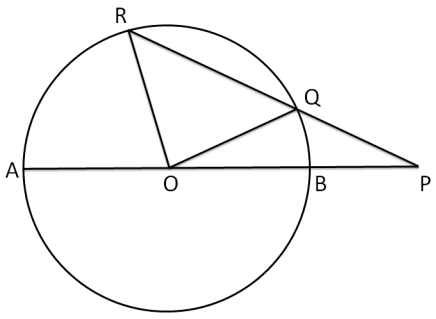
In triangle ABC, angle B is a right angle and X is the point on BC so that $BX:XC = 5:4$. Also, the length of AB is three times the length of CX and the area of triangle CXA is 54cm^2. Calculate the length of the perimeter of triangle CXA.



Level: Intermediate Ref No: M128 **Puzz Points: 13**

In the diagram, O is the centre of the circle and the straight lines AOBP and RQP meet at P. The length of PQ is equal to the radius of the circle. Prove that

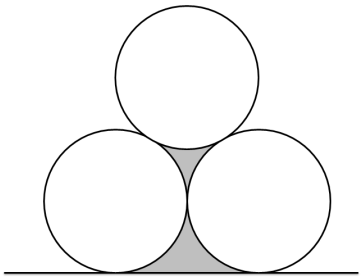
$$\angle AOR = 3 \times \angle BOQ$$



Level: Intermediate Ref No: M131 **Puzz Points: 15**

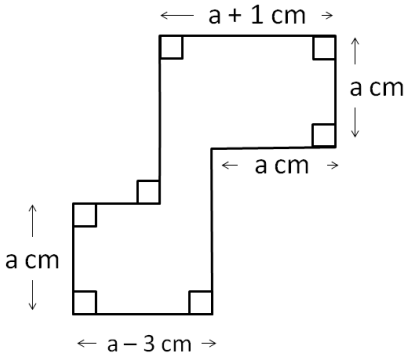
The region shown shaded in the diagram is bounded by three touching circles of radius 1 and the tangent to two of the circles.

Calculate the perimeter of the shaded region.



Level: Intermediate Ref No: M132 **Puzz Points: 15**

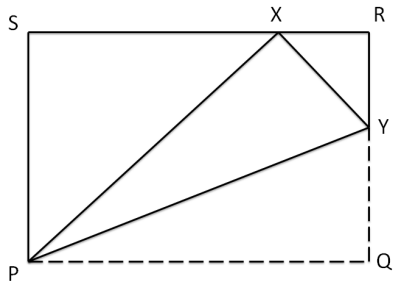
The shape shown in the diagram (not to scale) has a perimeter of length 72cm and an area equal to $147cm^2$. Calculate the value of a.



Level: Intermediate Ref No: M134 **Puzz Points: 18**

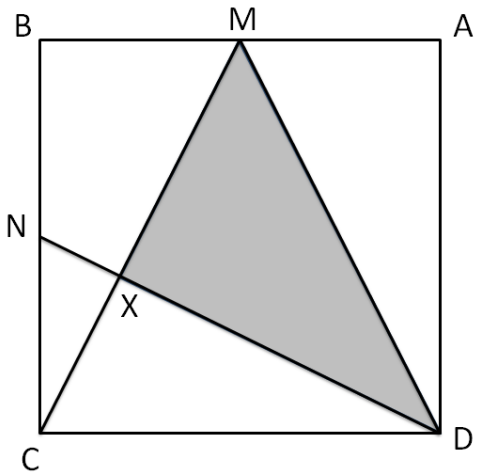
The rectangle $PQRS$ represents a sheet of A4 paper, which means that $PQ : PS = \sqrt{2} : 1$.

The rectangle is folded, as shown, so that Q comes to a point X on SR and the fold line PY passes though the corner P. Taking the length of PS to be 1 unit, find the lengths of the three sides of the triangle RXY.



Level: Intermediate Ref No: M140 **Puzz Points: 23**

In the diagram, X is the point of intersection of lines drawn from the corners C and D of square $ABCD$ to the midpoints M and N of sides AB and BC. Prove that the triangle MXD is right-angled with sides in the ratio $3:4:5$.



Level: Intermediate Ref No: M141 **Puzz Points: 23**

A sandcastle has a cylindrical base, on top of which is a second smaller cylinder, with a third even smaller cylinder on top. The three circular cylinders have the same height, and their radii are in the ratio $3:2:1$. The height of each cylinder is equal to the radius of the smallest cylinder.

Exactly 24 full buckets of sand were used to construct the sandcastle. The bucket is in the form of a frustum (part of a cone, as shown), whose larger radius equals its perpendicular height, and is twice its smaller radius.

Find the ratio of the total height of the sandcastle to that of the bucket.